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CERTAIN DYNAMIC MIXED PROBLEMS OF THE THEORY OF ELASTICITY

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V.M. ALEKSANDROV and V.G. BURIAK
(Rostov-on-Don, Voroshilovgrad)

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Vertical, horizontal and angular oscillations of a rigid stamp lying on an elastic, isotropic half-plane and acted by a harmonically varying load, are considered. The problems are formulated in such a manner, that one of the stress components is zero over the whole boundary of the half-plane. The problems for low, medium and high frequencies are reduced to integral equations, and three methods of solving these equations are given. Numerical solution of the problem of vertical oscillations shows that the approximate solutions obtained for the low, medium and high frequencies approach each other with an accuracy sufficient for practical applications. The problems for low frequencies were studied in [1–6], but half of the residue at the Rayleigh pole was not taken into account by the authors of [1,2,5,6].

1. Statement of the problems. Using the principle of limiting absorption [7] we reduce the problems of vertical (problem 1), horizontal (problem 2) and angular (problem 3) oscillations of a stamp to that of solving the following integral equations:

$$\int_{-1}^{1} q_\varepsilon(\xi) k_\varepsilon[\varkappa(\xi - x)]\, d\xi = 2\pi\Delta\delta_\varepsilon(x), \quad |x| \leqslant 1, \ \Delta = \frac{G}{a} \tag{1.1}$$

$$\varkappa = \omega a \sqrt{\frac{\rho}{G}}$$

$$q_\varepsilon(x) = q_{1\varepsilon}(x) + iq_{2\varepsilon}(x), \quad \delta_\varepsilon(x) = \delta_{1\varepsilon}(x) + i\delta_{2\varepsilon}(x)$$

(where for the problem 3 $\delta_\varepsilon(x) = \theta_\varepsilon x, \theta_\varepsilon = \theta_{1\varepsilon} + i\theta_{2\varepsilon}$ denotes the amplitude of the angle of rotation of the stamp). Here $\sigma(x, t) = q_\varepsilon(x)e^{i\omega t}$ denotes an unknown function of distribution of the contact stresses (normal for the problems 1 and 3, tangential for problem 2) under the stamp, $\delta_\varepsilon(x)e^{i\omega t}$ is the displacement of the stamp under applied load and $\delta_\varepsilon(x) = \delta_0(x)e^{-i\varphi}$ where φ is the phase difference between the oscillations of the stamp and the applied load generating these oscillations, $\delta_0(x)$ is the complex amplitude modulus of the oscillations of the stamp, ρ is density, G is the shear modulus of the elastic half-plane and $2a$ is the length of the line of contact. The x-axis is directed along the line of contact and the y-axis is perpendicular to the x-axis (see Fig. 1 of [5]).

In what follows we shall adopt, for the sake of brevity, the following convention. When speaking of the stresses, displacements and phase shift angles, we shall mean their amplitudes. Their true values can be obtained by multiplying by $e^{i\omega t}$.

The kernel $k_\varepsilon(x)$ of (1.1) has the form

$$k_\varepsilon(x) = \int_{-\infty}^{\infty} K_\varepsilon(u) e^{-i|x|u} du \tag{1.2}$$

$$K_\varepsilon(u) = \frac{\sqrt{u^2 - (1 - i\varepsilon) l^2}}{4u^2 \sqrt{u^2 - (1 - i\varepsilon) b^2} \sqrt{u^2 - (1 - i\varepsilon)} - [2u^2 - (1 - i\varepsilon)]^2} \tag{1.3}$$

$$b^2 = {}^1/_2 (1 - 2\nu) / (1 - \nu)$$

where $l = b$ for the problems 1 and 3 and $l = 1$ for the problem 2, ε is the proportionality coefficient characterizing the internal friction and ν is the Poisson's ratio. The function $K_\varepsilon(u)$ is even, has two poles and two branch points on the real axis symmetrically distributed about the coordinate origin.

2. **Low frequency method.** To find the uniform bound of the function $\delta_\varepsilon(x)$ as $\varepsilon \to 0$, we use the results of [8] to deform the contour of integration in (1.2) in such a manner that the poles and branch points of the function $K_\varepsilon(u)$ moving towards the real axis with $\varepsilon \to 0$ do not intersect this contour. As the result, the equations (1.1) and (1.2) become

$$\int_{-1}^{1} q(\xi) k[\varkappa(\xi - x)] d\xi = 2\pi\Delta\delta(x) \qquad \left(|x| \leqslant 1, \ \Delta = \frac{G}{a}\right) \tag{2.1}$$

$$k(x) = \int_\Gamma K(u) e^{-i|x|u} du \tag{2.2}$$

$$q(\xi) = \lim q_\varepsilon(\xi), \quad k(x) = \lim k_\varepsilon(x), \quad \delta(x) = \lim \delta_\varepsilon(x)$$
$$K(u) = \lim K_\varepsilon(u), \quad \varepsilon \to 0$$

The contour Γ in (2.2) coincides with the real axis, departing from it only to encircle the positive singularities from above, and the negative singularities from below. Let us write the integral appearing in (2.2), in the form

$$\int_\Gamma K(u) e^{-i|x|u} du = \int_\Gamma \left[K(u) - \frac{A_0}{u^2 - c_0^2}\right]^{-i|x|u} du + A_0 \int_\Gamma \frac{e^{-i|x|u} du}{u^2 - c_0^2} \tag{2.3}$$

where A_0 and c_0 are constants defined in [5] by formula (1.3). Taking due account of the fact that

$$\int_\Gamma \left[K(u) - \frac{A_0}{u^2 - c_0^2}\right] e^{-i|x|u} du = \int_{-\infty}^{\infty} \left[K(u) - \frac{A_0}{u^2 - c_0^2}\right] e^{-i|x|u} du$$

and computing the second integral in (2.3) by the method of residues, we obtain the following expression for the kernel (2.2):

$$k(x) = 2\int_0^\infty \left[K(u) - \frac{A_0}{u^2 - c_0^2}\right] \cos(xu) du - \frac{\pi i A_0}{c_0} e^{-i|x|c_0} \tag{2.4}$$

Here the last term in the right hand side represents half of the residue at the Rayleigh pole.

Writing (2.4) in the form of (1.3) of [5] and applying the methods given in the latter paper, we obtain the asymptotic formulas (1.15) of [5] in which

$$A_1 = D - \left(B + \frac{3}{2}E - E\ln\frac{2}{\varkappa}\right)\varkappa^2 + \left(-\frac{7}{2}C + \frac{103}{24}F + \frac{7}{2}F\ln\frac{2}{\varkappa} - \frac{5BE}{12D} - \frac{5E^2}{8D} + \frac{5E^2}{12D}\ln\frac{2}{\varkappa}\right)\varkappa^4, \quad B_1 = \left(2B + 3E - 2E\ln\frac{2}{\varkappa}\right)\varkappa^2 + \left(4C + \frac{7}{3}F - 4F\ln\frac{2}{\varkappa} + \frac{4BE}{3D} + \frac{2E^2}{D} - \frac{4E^2}{3D}\ln\frac{2}{\varkappa}\right)\varkappa^4 \tag{2.5}$$

$$C_1 = \left(4C + \frac{25}{3}F - 4F\ln\frac{2}{\varkappa} - \frac{2BE}{3D} - \frac{E^2}{D} + \frac{2E^2}{3D}\ln\frac{2}{\varkappa}\right)\varkappa^4$$

$$A_2 = -M\varkappa^2 - \left(\frac{7}{2}N + \frac{5ME}{12D}\right)\varkappa^4, \quad B_2 = 2M\varkappa^2 + 4\left(N + \frac{ME}{3D}\right)\varkappa^4$$

with the expressions for P_1, P_2, a_1, a_2 and a_3 remaining unchanged. Here for the problems 1 and 3 we have

$$\begin{aligned} A &= -0.08081, \quad D = 0.6999, \quad L = 1.09954 \\ B &= -0.2442, \quad E = -0.2575, \quad M = -0.4045 \\ C &= 0.03228, \quad F = 0.02190, \quad N = 0.03435 \end{aligned} \tag{2.6}$$

and for the problem 2

$$\begin{aligned} A &= 0.2819, \quad D = 0.6999, \quad L = 1.09956 \\ B &= -0.1490, \quad E = -0.1325, \quad M = -0.2081 \\ C &= 0.01416, \quad F = 0.009403, \quad N = 0.01477 \end{aligned} \tag{2.7}$$

3. **Medium frequency method.** Let us write the equations (1.1) and (1.2) in the form

$$\int_{-1}^{1} q_\varepsilon(\xi)\, k_\varepsilon[\varkappa(\xi - x)]\, d\xi = \pi\Delta\delta_\varepsilon(x) \quad \left(|x| \leqslant 1, \ \Delta = \frac{G}{a}\right) \tag{3.1}$$

$$k_\varepsilon(x) = \int_0^\infty [K_{1\varepsilon}(u) + K_{2\varepsilon}(u)]\cos(xu)\, du \tag{3.2}$$

$$K_{1\varepsilon}(u) = 4u^2(u^2 - \lambda^2)^{1/2}(u^2 - \lambda^2 b^2) / s_\varepsilon(u)$$

$$K_{2\varepsilon}(u) = 4(u^2 - \lambda^2 b^2)^{1/2}(u^2 - \lambda^2/2)^2 / s_\varepsilon(u) \quad \text{(problems 1 and 3)}$$

$$K_{1\varepsilon}(u) = 4(u^2 - \lambda^2 b^2)^{1/2}(u^2 - \lambda^2) / s_\varepsilon(u)$$

$$K_{2\varepsilon}(u) = 4(u^2 - \lambda^2)^{1/2}(u^2 - \lambda^2/2)^2 / s_\varepsilon(u) \quad \text{(problem 2)}$$

$$s_\varepsilon(u) = 16(1 - b^2)u^6 + 8\lambda^2(3b^2 - 3)u^4 + 8\lambda^4 u^2 - \lambda^6, \quad \lambda^2 = 1 - i\varepsilon$$

Applying to (3.1) the scheme used to obtain (1.1) in [9], we obtain the following functional equation:

$$\Phi_n^+(\alpha)[K_{1\varepsilon}(\alpha)+K_{2\varepsilon}(\alpha)] = \frac{a_0\Delta n!\, i^{n+1}}{\sqrt{2\pi}\alpha^{n+1}} + E^-(\alpha), \quad a_0 = 4(1-b^2) \tag{3.3}$$

where the quantities $\Phi_n^+(\alpha)$ and $E^-(\alpha)$ have the same meaning as $\Phi_n^+(\alpha)$, $E^-(\alpha)$ in (2.9) of [9].

Let us consider the system of functional equations

$$\Phi_{np}^{1+}(\alpha)K_{1\varepsilon}(\alpha) = \frac{a_0\Delta n!\, i^{n+1}}{2\sqrt{2\pi}\alpha^{n+1}} + \frac{1}{2}E_p^{1-}(\alpha) - \tag{3.4}$$

$$\Phi_{n,p-1}^{1+}(\alpha)K_{2\varepsilon}(\alpha) + \frac{1}{2}E_{p-1}^{2-}(\alpha)$$

$$\Phi_{np}^{2+}(\alpha)K_{2\varepsilon}(\alpha) = \frac{a_0\Delta n!\, i^{n+1}}{2\sqrt{2\pi}\alpha^{n+1}} + \frac{1}{2}E_p^{2-}(\alpha) -$$

$$\Phi_{n,p-1}^{2+}(\alpha)K_{1\varepsilon}(\alpha) + \frac{1}{2}E_{p-1}^{1-}(\alpha)$$

The system (3.4) which is equivalent to (3.3), can be solved using a method of consecutive approximations. For the zero approximation we have

$$\Phi_{n0}^{j+}(\alpha)K_{j\varepsilon}(\alpha) = \frac{a_0\Delta n!\, i^{n+1}}{2\sqrt{2\pi}\alpha^{n+1}} + \frac{1}{2}E_0^{j-}(\alpha), \quad j=1,2$$

$$K_{1\varepsilon}(u) = \frac{\sqrt{u^2-B_\varepsilon^2}\,(u^2-z_{1\varepsilon}^2)(u^2-z_{2\varepsilon}^2)}{(u^2-z_{3\varepsilon}'^2)(u^2-z_{1\varepsilon}'^2)(u^2-z_{2\varepsilon}'^2)}, \quad z_{2\varepsilon} = i\gamma$$

$$K_{2\varepsilon}(u) = \frac{\sqrt{u^2-D_\varepsilon^2}\,(u^2-E_\varepsilon^2)^2}{(u^2-z_{3\varepsilon}'^2)(u^2-z_{1\varepsilon}'^2)(u^2-z_{2\varepsilon}'^2)}, \quad E_\varepsilon = \frac{\lambda}{\sqrt{2}}$$

($B_\varepsilon = \lambda$, $z_{1\varepsilon} = D_\varepsilon = \lambda b$ for the problems 1 and 3, $B_\varepsilon = \lambda b$, $z_{1\varepsilon} = D_\varepsilon = \lambda$ for the problem 2, and $z_{1\varepsilon}'$, $z_{2\varepsilon}'$, $z_{3\varepsilon}'$ are the roots of the function $s_\varepsilon(u)$ lying in the upper half-plane).

Having performed for (3.4) the manipulations analogous to those carried out for (2.9) in [9] and taking into account the exact factorization of $K_{1\varepsilon}(u)$ and $K_{2\varepsilon}(u)$ we obtain the solution in the form

$$\varphi_{n0}^j(y) = \Delta\left[c_{1,n}^j\varphi_{*\varepsilon}^j(y) + \sum \frac{c_{k,n}^j}{(k-2)!}\int_0^y (y-u)^{k-2}\varphi_{*\varepsilon}^j(u)\,du\right] \tag{3.5}$$

$$c_{n+1,n}^1 = \frac{n!}{\sqrt{A_{1\varepsilon}}}, \quad c_{n+1,n}^2 = \frac{n!}{\sqrt{A_{2\varepsilon}}}, \quad \varphi_{*\varepsilon}^1(y) = -iA_{1\varepsilon}^{-1/2} \times$$

$$\operatorname{erf}\sqrt{-iB_\varepsilon y} + (\pi y)^{-1/2}e^{iB_\varepsilon y} + r_{1\varepsilon}e^{iz_{1\varepsilon}y}\operatorname{erf}\sqrt{i(z_{1\varepsilon}-B_\varepsilon)y} -$$

$$r_{2\varepsilon}e^{iz_{2\varepsilon}y}\operatorname{erf}\sqrt{i(z_{2\varepsilon}-B_\varepsilon)y}$$

$$\varphi_{*\varepsilon}^2(y) = -iA_{2\varepsilon}^{-1/2}\operatorname{erf}\sqrt{-iD_\varepsilon y} + r_{3\varepsilon}e^{iD_\varepsilon y} +$$

$$r_{4\varepsilon}e^{iE_\varepsilon y}\operatorname{erf}\sqrt{i(E_\varepsilon-D_\varepsilon)y}$$

$$r_{j\varepsilon} = i\frac{(z_{j\varepsilon} - z'_{1\varepsilon})(z_{j\varepsilon} - z'_{2\varepsilon})(z_{j\varepsilon} - z'_{3\varepsilon})}{z_{j\varepsilon}(z_{1\varepsilon} - z_{2\varepsilon})\sqrt{B_\varepsilon + iz_{j\varepsilon}}}, \quad j = 1, 2$$

$$r_{3\varepsilon} = \frac{1}{\sqrt{\pi y}}\left[1 - i\frac{A_{4\varepsilon}}{(E_\varepsilon - D_\varepsilon)}y\left(1 - \frac{z'_{3\varepsilon}}{E_\varepsilon}\right)\right]$$

$$r_{4\varepsilon} = \frac{1}{\sqrt{i(E_\varepsilon - D_\varepsilon)}}\left\{A_{3\varepsilon}\left(1 - \frac{z'_{3\varepsilon}}{E_\varepsilon}\right) + \right.$$

$$\left. A_{4\varepsilon}\left[y\left(1 - \frac{z'_{3\varepsilon}}{E_\varepsilon}\right) + \frac{z'_{3\varepsilon}(3E_\varepsilon - 2D_\varepsilon) - E_\varepsilon^2}{2iE_\varepsilon^2(E_\varepsilon - D_\varepsilon)}\right]\right\}$$

$$A_{1\varepsilon} = -\frac{iB_\varepsilon z_{1\varepsilon}^2 z_{2\varepsilon}^2}{z_{1\varepsilon}'^2 z_{2\varepsilon}'^2 z_{3\varepsilon}'^2}, \quad A_{2\varepsilon} = -\frac{iD_\varepsilon E_\varepsilon^4}{z_{1\varepsilon}'^2 z_{2\varepsilon}'^2 z_{3\varepsilon}'^2}$$

$$A_{3\varepsilon} = i(2E_\varepsilon - z'_{1\varepsilon} - z'_{2\varepsilon}), \quad A_{4\varepsilon} = -(E_\varepsilon - z'_{1\varepsilon})(E_\varepsilon - z'_{2\varepsilon})$$

where $\operatorname{erf} x$ is the probability integral.

The general solution of

$$\int_{-\infty}^{\infty} v_{j\varepsilon}(\xi)\, k_{j\varepsilon}[\varkappa(\xi - x)]\, d\xi = \pi\Delta\delta_\varepsilon(x), \quad |x| < \infty$$

$$k_{j\varepsilon}(x) = \int_0^\infty K_{j\varepsilon}(u)\cos(xu)\, du, \quad j = 1, 2$$

can be obtained using the formulas (2.31—(2.34) of [9], and has the form

$$v_{j\varepsilon}(x) = \varkappa\Delta a_0\left[\frac{\delta_\varepsilon(x)}{A_{j\varepsilon}} + \sum_{m=1}^{\infty}(-1)^m\frac{1}{\varkappa^{2m}}B_{jm}\delta^{(2m)}(x)\right]$$

Using the results of [9—12], we shall construct the approximate solutions of (1.1) in the form

$$q(x) = \lim_{\varepsilon,\gamma\to 0}\sum_{j=1}^{2}\varphi_{n0}^{j}[\varkappa(1 + x)]\,\varphi_{n0}^{j}[\varkappa(1 - x)]\,/\,v_{j\varepsilon}(x), \quad j = 1, 2 \tag{3.6}$$

noting that with the factorization taken into account,

$$K_{j\varepsilon}(\alpha) = K_{j\varepsilon}^{+}(\alpha)K_{j\varepsilon}^{-}(\alpha) \text{ when } \varepsilon\to 0, \quad \lambda\to -1$$

Construction of the higher approximations of (3.4) encounters difficulties in connection with computing integrals of the form

$$\frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic}\left\{\frac{a_0\Delta n!\, i^{n+1}}{2\sqrt{2\pi}\,\zeta^{n+1}}[K_{1\varepsilon}^{-}(\zeta)]^{-1} - \Phi_{n,p-1}^{1+}(\zeta)K_{2\varepsilon}(\zeta)[K_{1\varepsilon}^{-}(\zeta)]^{-1}\right\}\frac{d\zeta}{\zeta - \alpha}$$

We shall show however, that Φ_{n0}^{j+} can be determined with accuracy sufficient for practical purposes.

Having constructed in the above manner the approximate solutions $q_0(x)$ and $q_1(x)$ of (1.1) for medium $\varkappa$ in the form (3.6) for the case $\delta(x) = \delta$ (plane stamp) and $\delta(x) = \theta x$ (inclined stamp) respectively, we can find the force and moment for any nonplanar stamp using the formulas of [13]

$$P = \int_{-1}^{1} \delta(x)\, q_0(x)\, dx = P_1 + iP_2, \quad M = \int_{-1}^{1} \delta(x)\, q_1(x) dx = M_1 + iM_2 \qquad (3.7)$$

4. High frequency method. Let us write the kernel (2.2) of the integral equation (2.1) in a different form

$$k(x) = \int_{\Gamma} K(u)\, e^{-i\varkappa|x|u}\, du \qquad (4.1)$$

It can be shown that for $\varkappa \gg 1$ the kernel $k(x)$ can be written in the form

$$k(x) \approx -2\pi i\, [b_1\delta^\circ(x)/\varkappa + b_2 e^{-ic_0\varkappa|x|}] \qquad (4.2)$$
$$b_1 = i \lim_{u\to 0} K(u), \quad b_2 = \lim_{u\to c_0} (u - c_0)\, K(u)$$

Here $\delta^\circ(x)$ is the delta function, c_0 is the positive pole of the function $K(u)$ lying on the real axis, $b_1 = b$ for the problems 1 and 3 and $b_1 = 1$ for the problem 2. Substituting (4.2) into (2.1), we obtain

$$q(x) + \frac{b_2\varkappa}{b_1} \int_{-1}^{1} e^{-ic_0\varkappa|\xi - x|}\, q(\xi)\, d\xi = b_3\delta(x), \quad b_3 = \frac{i\varkappa\Delta}{b_1} \qquad (4.3)$$

Solving (4.3) by symbolic method proposed in [14], we obtain the following differential equation with constant coefficients:

$$q''(x) - \lambda_1^2 q(x) = b_3\, [\delta''(x) + c_0\varkappa^2\delta(x)], \quad \lambda_1 = \varkappa\, [c_0\, (2ib_2 / b_1 - c_0)]^{1/2} \qquad (4.4)$$

Approximating the function $\delta(x)$ by a polynomial yields a solution of (4.4) in the form

$$q(x) = q_0^*(x) + q_1^*(x), \quad q_0^*(x) = c_1 e^{\lambda_1 x} + c_2 e^{-\lambda_1 x} \qquad (4.5)$$

where $q_1^*(x)$ is a particular solution of the inhomogeneous equation (4.4). The arbitrary constants in the solution (4.5) can be found by substituting this solution into (4.3) and equating the coefficients of like powers of $e^{\lambda_1 x}$ and $e^{-\lambda_1 x}$.

5. Numerical study of problem 1. Let $\delta(x) = \delta = \text{const}$ $(\delta = \delta_1 + i\delta_2)$. Then the formula (3.6) and the first formula of (3.7) yield

$$q_0(x) = b_0\, [iQ(x)Q(-x) + S(x)S(-x)] \qquad (5.1)$$
$$Q(x) = [l_1 + l_2(1+x)]\, \mathrm{erf}\sqrt{i\varkappa(1+x)} + d_1\, [l_3 + l_4(1+x)] \times [\varkappa(1+x)]^{-1/2} e^{-i\varkappa(1+x)} + l_5 e^{-i\varkappa b(1+x)}\, \mathrm{erf}\sqrt{i\varkappa(1-b)(1+x)}$$
$$S(x) = [l_3 + d_2(1+x)][\varkappa(1+x)]^{-1/2} e^{-i\varkappa b(1+x)} + d_3\, \mathrm{erf}\sqrt{i\varkappa b(1+x)} + [d_4 + d_5(1+x)]\, e^{-i\varkappa h_1(1+x)}\, \mathrm{erf}\sqrt{i\varkappa(b - h_1)(1+x)}$$

$$P = P_1 + iP_2 = -\Delta\delta\,(1 - b^2)(J_1 + J_2) \tag{5.2}$$

where

$J_1 = (1 - c_0)^2(1 + in_1)^2 e^{-2\varkappa i} - im_1n_2\,[1 + im_2b^{-1} +$
$m_1\,(\varkappa - in_3)]e^{-2\varkappa bi} + {}^4/_3 ic_0{}^2m_2{}^2\varkappa^3 + 4c_0m_2n_4\varkappa^2 + \{4m_2c_0\,[2 -$
$c_0 + i(m_1n_5 - m_2n_6)] + 2ic_0{}^2\,(1 - im_1b^{-1})^2\}\varkappa +$
$m_2\,(1 - c_0{}^2)(m_2 - 2i) + c_0\,(2 - c_0) + 2m_1\,(m_2n_7 + in_8 +$
$m_1n_9)$

$J_2 = (b - c_0)^2 s_1{}^2 e^{-2\varkappa bi} + 2i\,(h_1 - b)^{-1}\{{}^4/_3 s_2m_4{}^2\varkappa^3 + 4m_4\,(m_3s_2 +$
$m_4h_1h_2)\varkappa^2 + 2\,[s_2s_3 + m_4\,(2m_3h_2h_1{}^{-1} + m_4h_3)]\varkappa + 2m_3s_2 +$
$s_3h_2h_1{}^{-1} + m_4\,(2m_3h_3 + im_4s_4)\}e^{-i\varkappa/h_1} + 2ic_0{}^2h_4b^{-1}\varkappa -$
$ic_0\,[is_5h_4 - 4bc_0\,(m_3 - is_3h_1{}^{-1} - 6m_3m_4 + 4im_4{}^2h_1{}^{-1})]b^{-2}$

$b_0 = -0.7143\varkappa,\quad l_1 = 0.7976 - 0.2133i,\quad l_2 = -0.5527i\varkappa$

$l_3 = 0.5642,\quad l_4 = 0.3118i\varkappa,\quad l_5 = 0.03252 + 0.1576i,\quad d_1 =$
$0.7071\,(1 - i)$

$d_2 = (-0.1282 + 0.3874i)\varkappa,\quad d_3 = -0.5714\,(1 + i),\quad d_4 =$
$0.5243 + 0.2223i$

$d_5 = (0.1350 + 0.2685i)\varkappa,\quad b = 0.5345,\quad c_0 = z_3' = 1.0783$

$h_1 = 0.7071$

$z_1' = 0.5211 + 0.05321i,\ z_2' = -0.5211 + 0.05321i,\ m_2 = ib^{-1}z_1'z_2'$

$m_1 = -ib^{-1}\,(z_1' + b)(z_2' + b),\quad m_3 = -i\,(z_1' + z_2' + h_1{}^{-1}),$

$m_4 = -(z_1' + h_1)(z_2' + h_1)$

$h_2 = -0.1315i,\quad h_3 = -15.115,\quad h_4 = (1 - im_3h_1{}^{-1} - 2m_4)^2$

$n_1 = m_1\,(1 - b)^{-1} + m_2,\quad n_2 = 4.4466,\quad n_3 = -2.6352$

$n_4 = c_0\,(i + m_1b^{-1}) + m_2\,(1 - {}^1/_2c_0),\quad n_5 = -1.9849,\quad n_6 =$
0.002842

$n_7 = -4.1474,\quad n_8 = 2.2104,\quad n_9 = 5.8752,\quad s_2 = 0.1378$

$s_1 = b^{-1}\,[1 - im_3\,(h_1 - b)^{-1} - m_4\,(h_1 - b)^{-2}],\quad s_3 = m_3{}^2 +$
$2m_4,\quad s_4 = 96.4547,\quad s_5 = -0.009224$

Here the value $\nu = 0.3$ was used in the computation.

Using the formulas (4.4), (4.5) and (3.7), we obtain the following expression for the case of $\delta\,(x) = \delta = \text{const.}$:

$$q_0\,(x) = H\,[R^{-1}\,\text{ch}\,(\lambda_1 x) + T],\quad P = 2H\,[(R\lambda_1)^{-1}\,\text{sh}\,\lambda_1 + T] \tag{5.3}$$

$H = -\dfrac{4c_0\varkappa^2b_2\Delta\delta}{b\,(2b_2 + ic_0b)},\quad R = (ic_0\varkappa + \lambda_1)\,e^{\lambda_1} +$
$(ic_0\varkappa - \lambda_1)\,e^{-\lambda_1},\quad T = \dfrac{b}{4\varkappa b_2}$

where λ_1 can be found from the second formula of (4.4) and $b_2 = 0.1661$.

The table below gives the results of computing the quantities $P_1^* = P_1 / \Delta$, $P_2^* = P_2 / \Delta$ by means of the asymptotic formulas (2.5) and (5.2) and the second formula of (5.3), for the low, medium and high values of $\varkappa$, with $(P_1^* = -\alpha\delta_1 - \beta\delta_2,\ P_2^* = \beta\delta_1 - \alpha\delta_2)$.

η	Eq. (2.5)		Eq. (5.2)		Eq. (5.3)	
	α	β	α	β	α	β
0.25	1.37	0.962	1.65	0.436	0.0194	0.812
0.50	1.55	1.44	1.98	1.01	0.120	1.45
0.75	1.66	1.74	2.37	1.78	0.324	1.99
1.00	1.52	1.77	2.72	2.80	0.636	2.49
1.25	0.964	1.48	2.91	4.05	1.06	2.99
1.50	0.329	1.17	2.82	5.43	1.56	3.54
1.75	—0.0863	0.972	2.40	6.79	2.12	4.18
2.00	—0.317	0.864	1.66	7.95	2.67	4.91
2.25	—0.444	0.800	0.707	8.67	3.17	5.72

The figure depicts the dependence of δ_0^* and φ on the dimensionless frequency $\varkappa$ for various values of the dimensionless mass M^* obtained from the formulas (2.4) of [5]. It is clear that the modulus of the complex amplitude δ_0^* decreases and the phase shift increases with increasing $\varkappa$ and M^*, and this fully agrees with the physical aspects of the problem.

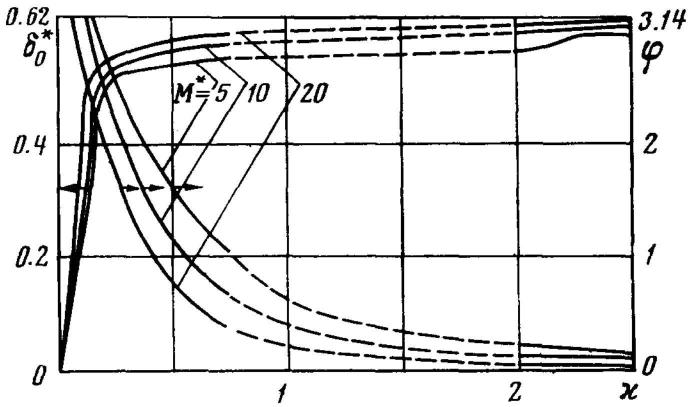

Fig. 1

Thus the computations have shown that the approximate solutions converge, in the range $0.5 \leqslant \varkappa \leqslant 1$ for the low and medium frequencies and in the range $1.5 \leqslant \varkappa \leqslant 2.25$ for the medium and high frequencies. This makes feasible the study of all basic characteristics of the problem for any value of the parameter $\varkappa$.

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